Marginal hyperchaos synchronization with a single driving variable

ELMAN M. SHAHVERDIEV,[1]
Institute of Physics of Academy of Sciences of Azerbaijan,
33,H.Javid Av., 370143-Baku,Azerbaijan.

The seminal papers by Pecora and Carrol (PC) [1] and Ott, Grebogi and Yorke (OGY) [2] in 1990 have induced avalanche of research works in the field of chaos control. Chaos synchronization in dynamical systems is one of methods of controling chaos, see, e.g. [1-8] and references therein.The interest to chaos synchronization in part is due to the application of this phenomenen in secure communications, in modeling of brain activity and recognition processes,etc [1-8]. Also it should be mentioned that this method of chaos control may result in improved performance of chaotic systems [1-8]. According to PC [1] synchronization of two systems occurs when the trajectories of one of the systems will converge to the same values as the other and they will remain in step with each other. For the chaotic systems synchronization is performed by the linking of chaotic systems with a common signal or signals (the so-called drivers): suppose that we have a chaotic dynamical system of three or more state variables. In the above mentioned way of chaos control one or some of these state variables can be used as an input to drive a subsystem consisting of remaining state variables and which is a replica of part of the original system.In [1] it has been shown that if the real parts of the Lyapunov exponents for the subsystem (below: sub-Lyapunov exponents) are negative then the subsystem synchronizes to the chaotic evolution of original system. If the largest sub-Lyapunov exponent is not negative, then one can use the nonreplica approach to chaos synchronization [9]. Within the nonreplica approach to chaos synchronization one can try to perform chaos synchronization between the original chaotic system and nonreplica response system with control terms vanishable upon synchronization.To be more specific, one can try to make negative the real parts of the conditional Lyapunov exponents of the nonreplica response system. As it has been shown in [9] from the application viewpoint nonreplica approach has some advantages over the replica one.

Recently in [10] it has been indicated that for more secure communication purposes the use of hyperchaos is more reliable. Quite naturally in the light of this result the investigation of hyperchaos synchronization is of paramount importance. According to Pyragas for hyperchaos synchronization at least two drive variables are needed [11].

Recently this idea was challenged in [12] in the sense that instead of several driving variables one can try to drive the response system with a scalar combination of those driving variables. But one should keep in mind that in this case the synchronization occurs between the nonreplica system and original chaotic system.Recent paper [13] also falls into this category, although its authors are using only single control term added to the replica response system.

In recent work [14] the classification of different types of synchronization is conducted. Such a classifiation into different types corresponds to the different values for the sub- (or conditional) Lyapunov exponents and still there is no unique generally accepted classification. For example, according to [15] if one of sub- Lyapunov exponents is equal to zero, while others are negative, then one can still speak of synchronization between the response and drive systems in the general sense: a generalized synchronization introduced for drive-response systems is defined as the presence of some functional relation between the states of response and drive. According to [14], the similar situation could be characterized by the so-called marginal synchronization:there are there types of marginal synchronization:1) marginal constant synchronization: in this case the response system becomes synchronized with the drive, but with a constant separation.

2) marginal oscillatory synchronization: this type of synchronization implies that the difference between the drive and response will change in an oscillatory fashion with a frequency that will depend on the imaginary part and with constant amplitude that will be related to the difference

[1]e-mail:shahverdiev@lan.ab.az

at the moment in which the connection starts.
3) sized synchronization: in this type of synchronization also one has a single zero sub-Lyapunov exponent; in this case the observed behavior is different from the case of marginal constant synchronization and consists in that the response system exhibits the same qualitative behavior as the drive, but with different size (and sometimes with different symmetry); as a prominent example of this type of synchronization one can cite the case z driving for the classical Lorenz model [7]. It is easy to show that one of sub- Lyapunov exponents for the Lorenz model in the case of z driving really is equal to zero. By simple calculations one can easily obtain the following equation for the sub-Lyapunov exponents:

$$\lambda^2 + \lambda(\sigma + 1) - \sigma(r - z) = 0, \qquad (1)$$

where $z = z(t)$ is the solution of the original Lorenz system. As it has been shown in [16], for those dynamical systems, whose chaotic behavior has arisen out of instability of the steady state solutions (fixed points) while calculating the sub-Lyapunov exponents one can replace the time dependent solutions of the dynamical systems with the steady state (st) solutions safely. As the Lorenz model has the above- mentioned property, and $z^{st} = r - 1$, one can easily establish that one of the sub-Lyapunov exponents is equal to zero.
In the above mentioned papers [14-15] the presented examples represent third-order nonlinear dynamical systems.
In this paper we present an example of marginal or general type synchronization in higher dimensional system, to be more specific in one of four dimensional hyperchaos Rössler models:

$$\frac{dx}{dt} = -y - z - w,$$
$$\frac{dy}{dt} = x, \qquad (2)$$
$$\frac{dz}{dt} = a(y - y^2) - bz,$$
$$\frac{dw}{dt} = c(\frac{z}{2} - z^2) - dw,$$

According to [17,18], nonlinear system (1) exhibits hyperchaotic behavior with some positive values of system's parameters a, b, c, d. First consider as a driver state variable x. Then the response system could be written in the following form:

$$\frac{dy_r}{dt} = x = A_1,$$
$$\frac{dz_r}{dt} = a(y_r - y_r^2) - bz_r = A_2, \qquad (3)$$
$$\frac{dw_r}{dt} = c(\frac{z_r}{2} - z_r^2) - dw_r = A_3,$$

The eigenvalues of the Jacobian of (3) are to be found from the equation:

$$\lambda(\lambda + b)(\lambda + d) = 0, \qquad (4)$$

In other words, in the case of x driving, according to classification of [14] marginal constant synchronization is possible, as one of sub-Lyapunov exponents is negative, while others -positive.

Surprisingly, due to the form of the nonlinear system under study in the case of y driving we obtain exactly the same equation for the sub-Lyapunov exponents. So the marginal constant synchronization takes place in the case of y driving too.
As the investigations show quite different type of marginal synchronization, namely marginal oscillatory synchronization could be realized in the case of z driving. Indeed, as the calculations indicate in this case the conditional Lyapunov exponents satisfy the equation:

$$(\lambda + d)(\lambda^2 + 1) = 0, \tag{5}$$

According to the classification in [14], this case quite "eligible" to be named as the marginal oscillatory synchronization, as one of sub- Lyapunov exponents is negative, while the two others are complex conjugate with zero real parts.So far we considered the cases of driving with x, y, z variables and we have succeeded in marginal synchronization of hyperchaos only using the fact of positiveness of part of the system's parameters, namely b, d. As the calculations show the case of w driving is a bit more complicated in the sense that some additional relationships between the system's parameters are required. Depending on these relationships different types of synchronization, according to the classification of [14] are also possible.
Thus in this paper for the first time (to our knowledge) we have demonstrated the possibility of hyperchaos synchronization with a single driving variable within the replica approach.

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